DESIGN OF NON-RECURSIVE AND RECURSIVE DIGITAL BAND PASS FILTERS FOR GENERAL PURPOSE APPLICATIONS

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SUMMARY: *This work is based on the study of non-recursive and recursive digital band pass filters for an audio equalizer. The design of FIR (non-recursive) and IIR (recursive) filters were made following the design specifications for this application. To find the solution of this problem, first we will define the filter characteristics to find the respective coefficients for both types of filter. Then, we will deduce the expression of H(z) and the structure diagram of the filters with their correspondent coefficients. Next, we will deduce the expression of the difference equations that can be used for programing the algorithms for the simulation of the filters in a DSP microprocessor. Finally we will present the response of magnitude and phase of the designed filters complying with the required design characteristics. Whole design was simulated using the software Mathcad.*

RESUMEN: *Este trabajo está basado en el estudio de filtros digitales pasa banda no-recursivos y recursivos para un ecualizador de audio. El diseño de filtros FIR (no-recursivo) e IIR (recursivo) fueron hechos siguiendo las especificaciones de diseño para esta aplicación. Para hallar la solución de este problema, primero vamos a definir las características del filtro para hallar los coeficientes respectivos para ambos tipos de filtro. Luego, deduciremos la expresión H(z) y el diagrama de estructura de los filtros con sus coeficientes correspondientes. A continuación, deduciremos la expresión de la ecuación diferencial que se usará para programar los algoritmos para la simulación de los filtros en un microprocesador DSP. Finalmente se presentará la respuesta en magnitud y fase de los filtros diseñados cumpliendo con las características de diseño requeridas. Todo el diseño fue simulado usando el software Mathcad.*

1 INTRODUCTION

This report presents the study of recursive and non-recursive filter, which are used on many applications. The design methods for each of these two classes of filters are different because of their distinctly different properties.

In non-recursive filter structures the output depends only on the input, where we have feed-forward paths. The FIR filter has a finite memory and can have excellent linear phase characteristics, but it requires a large number of terms, to obtain a relatively sharp cutoff frequency response.

In recursive filter structures the output depends both on the input and on the previous outputs, where we have both feed-forward and feed-back paths. The IIR filter has an infinite memory and tends to have fewer terms, but its phase characteristics are not as linear as FIR.

Filter design can be implemented in the time-domain or frequency-domain. In this case, we shall be dealing with the design of filters specified in the frequency-domain because it is more suitable and precise for us. We will use

2 PRESENTATION OF THE PROBLEM

The following problem requires 2 designs. First we require designing a non-recursive digital band pass filter with the following characteristics:

Sampling Frequency	1000 KHz
Number of coefficients	51
Band Width	100 KHz
Centre Frequency	250 KHz

(For Hamming, Von Hann and Kaiser Beta=4 and Beta=8 filters)

Second we require designing a recursive digital band pass filter with the following characteristics:

Sampling Frequency	1000 KHz
Range of the pass band (ripple free attenuation between 0 and 3 dB)	From 200 to 300 KHz
Attenuation must be at least 30 dB for frequencies	Less than 150 KHz and more than 350 KHz

(For Butterworth and Chebyshev I filters)

3 DESCRIPTION OF THE SOLUTION

<u>FIR Filters Design (non-recursives filters)</u>

First of all, we define the filter characteristics:

$$Fs := 1000000 \quad Bw := 100000 \quad Fc := \frac{Bw}{2}$$

$$M := 51 \quad Fo := 250000$$

$$Fo + Fc - (Fo - Fc) = 100000 \qquad N := \frac{M-1}{2}$$

$$N = 25 \qquad n := 0..M-1$$

$$h_n := \left(2\cdot\frac{Fc}{Fs}\right)\cdot\left[\frac{\sin\left[(n-N)\cdot 2\cdot\pi\cdot\frac{Fc}{Fs}\right]}{\left[(n-N)\cdot 2\cdot\pi\cdot\frac{Fc}{Fs}\right]}\right]$$

$$h_n := \mathrm{if}\left(n = N, 2\cdot\frac{Fc}{Fs}, h_n\right)$$

Then, we define the corresponding window to find the coefficients for each filter. For each filter, we have to convert the low pass to band pass.

- ***<u>Hamming</u>***

$$Whm_n := 0.54 + (1 - 0.54)\cdot\cos\left[(n-N)\cdot\frac{\pi}{N}\right]$$

$$Wrhm(n) := h_n \cdot Whm_n$$

$$WrhmBP(n) := 2\cdot\cos\left[(n-N)\cdot 2\cdot\pi\cdot\frac{Fo}{Fs}\right]\cdot Wrhm(n)$$

- ***<u>Hanning</u>***

$$Whn_n := 0.5 + (1 - 0.5)\cdot\cos\left[(n-N)\cdot\frac{\pi}{N}\right]$$

$$Wrhn(n) := h_n \cdot Whn_n$$

$$WrhnBP(n) := 2\cdot\cos\left[(n-N)\cdot 2\cdot\pi\cdot\frac{Fo}{Fs}\right]\cdot Wrhn(n)$$

- ***<u>Kaiser β = 4</u>***

$$j := 1..15 \qquad \beta := 4$$

$$x_n := \beta\cdot\sqrt{\left[1 - \left(\frac{n-N}{N}\right)^2\right]} \qquad Io_n := 1 + \sum_j \left[\frac{\left(\frac{x_n}{2}\right)^j}{j!}\right]^2$$

$$Wk4_n := \frac{Io_n}{Io_N} \qquad Wrk4(n) := h_n \cdot Wk4_n$$

$$Wrk4BP(n) := 2\cdot\cos\left[(n-N)\cdot 2\cdot\pi\cdot\frac{Fo}{Fs}\right]\cdot Wrk4(n)$$

- ***<u>Kaiser β = 8</u>***

$$j := 1..15 \qquad \beta := 8$$

$$x_n := \beta\cdot\sqrt{\left[1 - \left(\frac{n-N}{N}\right)^2\right]} \qquad Io_n := 1 + \sum_j \left[\frac{\left(\frac{x_n}{2}\right)^j}{j!}\right]^2$$

$$Wk8_n := \frac{Io_n}{Io_N} \qquad Wrk8(n) := h_n \cdot Wk8_n$$

$$Wrk8BP(n) := 2\cdot\cos\left[(n-N)\cdot 2\cdot\pi\cdot\frac{Fo}{Fs}\right]\cdot Wrk8(n)$$

With these formulas, we find the coefficients (this result is shown in Part 4).

<u>IIR Filters Design (recursives filters)</u>

First of all, we define the parameters specified in the filter characteristics.

$Fs := 1000000 \quad fd1 := 200000 \quad Fd1 := 300000$

$fd2 := 150000 \quad Fd2 := 350000 \quad Att := 30$

$$T := \frac{1}{Fs} \qquad fd0 := \frac{Fd1 + fd1}{2}$$

$$\beta := \cot\left[\left(\frac{Fd1 - fd1}{2 \cdot Fs}\right) \cdot 2 \cdot \pi\right]$$

$$\alpha := \frac{\cos\left(\frac{Fd1 + fd1}{2 \cdot Fs} \cdot 2 \cdot \pi\right)}{\cos\left(\frac{Fd1 - fd1}{2 \cdot Fs} \cdot 2 \cdot \pi\right)} \qquad \alpha = 0$$

$wd2T := 2 \cdot \pi \cdot fd2 \cdot T \quad wd1T := 2 \cdot \pi \cdot fd1 \cdot T$

$wd2T = 0.942477796 \quad wd1T = 1.256637061$

$Wd1T := 2 \cdot \pi \cdot Fd1 \cdot T \quad Wd2T := 2 \cdot \pi \cdot Fd2 \cdot T$

$Wd1T = 1.884955592 \quad Wd2T = 2.199114858$

$wa2 := -\beta \cdot \cot(wd2T) \quad wa1 := -\beta \cdot \cot(wd1T)$

$Wa1 := -\beta \cdot \cot(Wd1T) \quad Wa2 := -\beta \cdot \cot(Wd2T)$

$$\frac{wa2}{wa1} = 2.23606797 \qquad \frac{Wa2}{Wa1} = 2.236067977$$

Then, using the analog models of the Butterworth and Chebyshev I filters and the bilinear transformation principles, we will deduce the correspondent poles of the filters.

- ***Butterworth Filter***

Taking the analog Butterworth filter model, that responds to the expression:

$$H\ w\ ^2 = \frac{1}{1 + \frac{w}{w_c}\ ^{2n}} \quad or$$

$$H\ s\ .H\ -s\ = \frac{1}{1 + (\frac{-s^2}{w_c{}^2})^n}$$

So, we have to find the value of "n", so that from Fd2 Hz, the signal attenuate Att dB.

$$n := \text{root}\left[10 \cdot \log\left[\frac{1}{1 + \left(\frac{Wa2}{Wa1}\right)^{2 \cdot n}}\right] + Att, n\right]$$

$n = 4.291$

$n := \text{ceil}(n)$

$n = 5$

n=ceil(n) was chosen, because "n" have to be integer, leaving the expression: (the first comes from the second)

$$H\ w\ ^2 = \frac{1}{1 + \frac{w}{w_c}\ ^{2n}} \quad or$$

$$H\ s\ .H\ -s\ = \frac{1}{1 + (\frac{-s^2}{w_c{}^2})^n}$$

So, we have to separate H(s), this means to find the roots of (-s^2/wc^2)-(-1)^(1/n):

$k := 0 .. n - 1$

$$s_k := \exp\left(i \cdot \frac{\pi}{2}\right) \cdot \exp\left[i \cdot (2 \cdot k + 1) \cdot \frac{\pi}{2 \cdot n}\right]$$

$$s_k = \begin{pmatrix} -0.309016994 + 0.951056516i \\ -0.809016994 + 0.587785252i \\ -1 \\ -0.809016994 - 0.587785252i \\ -0.309016994 - 0.951056516i \end{pmatrix}$$

S_K shows the correspondent poles of the Butterworth Filter.

- ***Chebyshev I Filter***

Now, taking the analog Butterworth filter model, that responds to the expression:

$$H\ w\ ^2 = \frac{1}{1 + \frac{w}{w_c}\ ^{2n}} \quad or$$

$$H\ s\ .H\ -s\ = \frac{1}{1 + (\frac{-s^2}{w_c{}^2})^n}$$

So, we have to find the value of "n", so that from Fd2 Hz, the signal attenuate Att dB.

$$Att := 30 \qquad \Omega c := Wa1 \qquad \varepsilon := 1$$

$$\Omega s := Wa2 \qquad \delta 1 := \frac{1}{\sqrt{1+\varepsilon}} \qquad \delta 2 := 10^{\frac{-Att}{20}}$$

$$\delta 1^2 = 0.5$$

$$\delta 2^2 = 0.001 \qquad 10^{\frac{-Att}{20}} = 0.0316227766$$

$$N := \frac{\log\left[\frac{\left[\sqrt{1-\delta 2^2}+\sqrt{1-\delta 2^2\cdot\left(1+\varepsilon^2\right)}\right]}{\varepsilon\cdot\delta 2}\right]}{\log\left[\frac{\Omega s}{\Omega c}+\sqrt{\left(\frac{\Omega s}{\Omega c}\right)^2-1}\right]}$$

$$N := \text{ceil}(N) \qquad N = 3$$

$$\beta := \left(\frac{\sqrt{1+\varepsilon^2}+1}{\varepsilon}\right)^{\frac{1}{N}} \qquad r1 := \Omega c\cdot\frac{\beta^2+1}{2\beta}$$

$$r2 := \Omega c\cdot\frac{\beta^2-1}{2\beta}$$

$$k := 0..N-1 \qquad n := N$$

$$s_k := r2\cdot\cos\left[\frac{\pi}{2}+\frac{[(2\cdot k+1)\cdot\pi]}{2\cdot N}\right] + j\, r1\cdot\sin\left[\frac{\pi}{2}+\frac{[(2\cdot k+1)\cdot\pi]}{2\cdot N}\right]$$

$$s_k = \begin{pmatrix} -0.1490179095+0.9036697472i \\ -0.298035819 \\ -0.1490179095-0.9036697472i \end{pmatrix}$$

S_K shows the correspondent poles of the Chebyshev I Filter.

4 RESULTS

FIR Filters Results

Applying the formulas listed above; there are 51 coefficients, from 0 to 50. We are going to show, the coefficients, the expression of H(z), the structure diagram and the algorithm for the simulation for each of the windows. Finally, we are going to show simultaneously, the magnitude and phase response for all these 4 filters.

- ***Hamming***

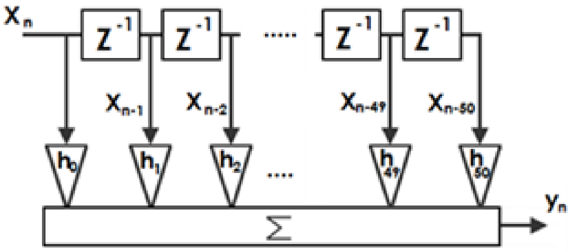
The coeffcientes for this filter are:

$$n := 0..N$$

n =	M − 1 − n	WrhmBP(n) =
0	50	0
1	49	0.002109711
2	48	0
3	47	-0.001910147
4	46	0
5	45	0
6	44	0
7	43	0.005130326
8	42	0
9	41	-0.013022775
10	40	0
11	39	0.019625797
12	38	0
13	37	-0.017739485
14	36	0
15	35	0
16	34	0
17	33	0.036787196
18	32	0
19	31	-0.088329305
20	30	0
21	29	0.14275282
22	28	0
23	27	-0.184393966
24	26	0
25	25	0.2

The expression of H(z) (that correspond to the filter), the structure diagram, and the algorithm for the simulation are:

$$H1(z) := \sum_{n=0}^{50}\left(WrhmBP(n)\cdot z^{-n}\right)$$



$$h(n) := WrhmBP(n)$$

$$y(n) := \sum_{k=0}^{50} (h(n) \cdot x(n-k))$$

- ***Hanning***

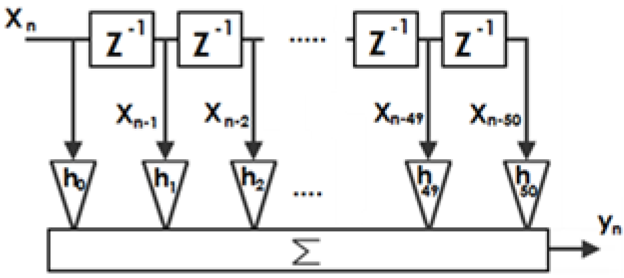
The coeffcientes for this filter are:

$n := 0..N$

n =	M − 1 − n	WrhnBP(n) =
0	50	0
1	49	0.000099463
2	48	0
3	47	-0.000597212
4	46	0
5	45	0
6	44	0
7	43	0.003768733
8	42	0
9	41	-0.01086464
10	40	0
11	39	0.017571758
12	38	0
13	37	-0.016570486
14	36	0
15	35	0
16	34	0
17	33	0.035918738
18	32	0
19	31	-0.087235311
20	30	0
21	29	0.142003905
22	28	0
23	27	-0.184158845
24	26	0
25	25	0.2

The expression of H(z) (that correspond to the filter), the structure diagram, and the algorithm for the simulation are:

$$H2(z) := \sum_{n=0}^{50} \left(WrhnBP(n) \cdot z^{-n} \right)$$



$$h(n) := WrhnBP(n)$$

$$y(n) := \sum_{k=0}^{50} (h(n) \cdot x(n-k))$$

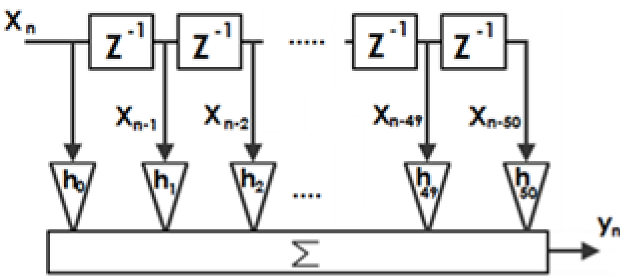
- ***Kaiser β = 4***

$n := 0..N$

$n =$	$M-1-n$	$Wrk4BP(n) =$
0	50	0
1	49	0.002988979
2	48	0
3	47	-0.003201926
4	46	0
5	45	0
6	44	0
7	43	0.007502119
8	42	0
9	41	-0.017347817
10	40	0
11	39	0.024166385
12	38	0
13	37	-0.020513105
14	36	0
15	35	0
16	34	0
17	33	0.03904048
18	32	0
19	31	-0.09124489
20	30	0
21	29	0.14478487
22	28	0
23	27	-0.185038538
24	26	0
25	25	0.2

The expression of H(z) (that correspond to the filter), the structure diagram, and the algorithm for the simulation are:

$$H3(z) := \sum_{n=0}^{50} \left(Wrk4BP(n) \cdot z^{-n}\right)$$



$$h(n) := WrhnBP(n)$$

$$y(n) := \sum_{k=0}^{50} (h(n) \cdot x(n-k))$$

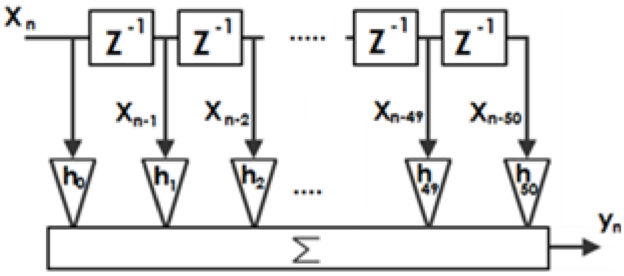
- ***Kaiser β = 8***

$n := 0..N$

$n =$	$M-1-n$	$Wrk8BP(n) =$
0	50	0
1	49	0.000159729
2	48	0
3	47	-0.000378522
4	46	0
5	45	0
6	44	0
7	43	0.0021754
8	42	0
9	41	-0.006805724
10	40	0
11	39	0.012094768
12	38	0
13	37	-0.012504134
14	36	0
15	35	0
16	34	0
17	33	0.031585434
18	32	0
19	31	-0.081108901
20	30	0
21	29	0.137462699
22	28	0
23	27	-0.182665041
24	26	0
25	25	0.2

The expression of H(z) (that correspond to the filter), the structure diagram, and the algorithm for the simulation are:

$$H4(z) := \sum_{n=0}^{50} \left(Wrk8BP(n) \cdot z^{-n}\right)$$



$$h(n) := \mathrm{Wrk8BP}(n)$$

$$y(n) := \sum_{k=0}^{50} (h(n)\cdot x(n-k))$$

Finally, we will compare the magnitude and phase response of the 4 filters designed, but before that, we will show the correspondent formulas to get these responses for some frequencies.

We define the variable F, shown as:

$$F := 0, \frac{Fs}{400} .. \frac{Fs}{2}$$

The deduced expressions of amplitude and phase are:

$$\mathrm{Hrhm}(F) := \left(e^{-i\cdot N\cdot 2\cdot\pi\cdot\frac{F}{Fs}}\right)\cdot\left[\mathrm{WrhmBP}(N) + 2\cdot\left[\sum_{n=0}^{N-1}\left[\mathrm{WrhmBP}(n)\cdot\cos\left[(N-n)\cdot 2\cdot\pi\cdot\frac{F}{Fs}\right]\right]\right]\right]$$

$$\mathrm{HrhmdB}(F) := 10\cdot\log\left[\left(\left|\frac{\mathrm{Hrhm}(F)}{\mathrm{Hrhm}(Fo)}\right|\right)^2\right]$$

$$\phi\mathrm{Hrhm}(F) := \arg(\mathrm{Hrhm}(F))\cdot\frac{180}{\pi}$$

Now calculating some attenuation for the different windows:

- ***Hamming***

HrhmdB(0) = −53.904768059	HrhmdB(300000) = −6.010526659
HrhmdB(50000) = −92.917624668	HrhmdB(350000) = −62.60372353
HrhmdB(100000) = −52.56969566	HrhmdB(400000) = −52.56969566
HrhmdB(150000) = −62.603723531	HrhmdB(450000) = −92.91762466
HrhmdB(200000) = −6.010526659	HrhmdB(500000) = −53.90476805
HrhmdB(250000) = 0	

- ***Hanning***

HrhndB(0) = −77.891295424	HrhndB(300000) = −6.049149209
HrhndB(50000) = −96.694812763	HrhndB(350000) = −58.065751378
HrhndB(100000) = −66.564148612	HrhndB(400000) = −66.564148612
HrhndB(150000) = −58.065751378	HrhndB(450000) = −96.694812763
HrhndB(200000) = −6.049149209	HrhndB(500000) = −77.891295424
HrhndB(250000) = 0	

- ***Kaiser β = 4***

Hrk4dB(0) = −52.811286733	Hrk4dB(300000) = −5.936322301
Hrk4dB(50000) = −68.958986828	Hrk4dB(350000) = −48.856513555
Hrk4dB(100000) = −51.155966475	Hrk4dB(400000) = −51.155966475
Hrk4dB(150000) = −48.856513555	Hrk4dB(450000) = −68.958986828
Hrk4dB(200000) = −5.936322301	Hrk4dB(500000) = −52.811286733
Hrk4dB(250000) = 0	

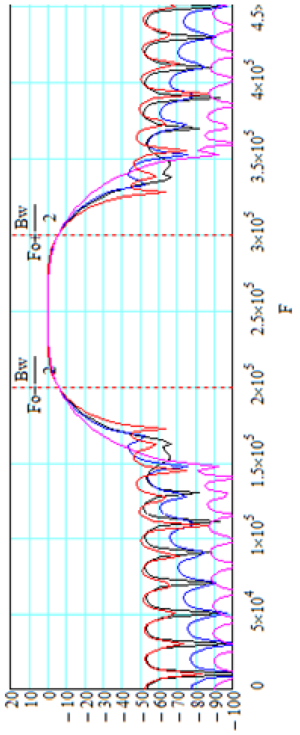
- ***Kaiser β = 8***

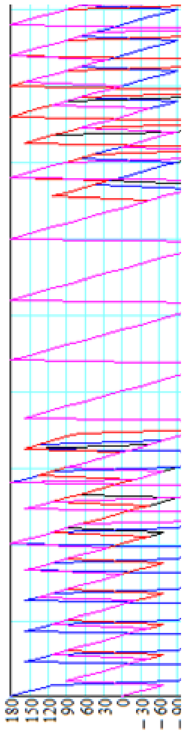
Hrk8dB(0) = −90.053832852	Hrk8dB(300000) = −6.017256184
Hrk8dB(50000) = −92.845542603	Hrk8dB(350000) = −72.30269479
Hrk8dB(100000) = −98.736663889	Hrk8dB(400000) = −98.736663889
Hrk8dB(150000) = −72.30269479	Hrk8dB(450000) = −92.845542603
Hrk8dB(200000) = −6.017256184	Hrk8dB(500000) = −90.053832852
Hrk8dB(250000) = 0	

Now, we'll represent the response of magnitude and phase of all these filters:

- The black graphic shows the hamming filter response.
- The blue graphic shows the hanning filter response.
- The red graphic shows the Kaiser β=4 filter response.
- The pink graphic shows the Kaise β=8 filter response.

From Figure 1, we can see that all the filters follow the characteristics specified for the design, we can see the cut frequencies and the stop frecuencies.





$$\beta := \cot\left[\left(\frac{Fd1 - fd1}{Fs}\right)\cdot\pi\right]$$

$$H(z) := \frac{1}{\prod_{l=0}^{n-1}\left[\left(\beta\cdot\frac{z^{-2} - 2\cdot\alpha\cdot z^{-1} + 1}{1 - z^{-2}}\right) - s_l\right]}$$

Then, we have to define many equations to find the coefficients:

$$i := 0, 1 .. 2000 \qquad \sigma := 0.0000000001$$

$$ESC := 1 \qquad F_i := \frac{i}{2000}\cdot Fs\cdot ESC$$

$$S_i := \sigma + i\cdot 2\cdot\pi\cdot F_i \qquad Z_i := \exp(S_i\cdot T)$$

$$HS_i := \frac{1}{\prod_{l=0}^{n-1}(S_i - s_l)} \qquad HSDB_i := 20\cdot\log\left(\left|\frac{HS_i}{HS_0}\right|\right)$$

$$HSDB(i) := HSDB_i$$

$$HD0Z_i := \frac{1}{\prod_{l=0}^{n-1}\left[\beta\cdot\frac{(Z_i)^{-2} - 2\cdot\alpha\cdot(Z_i)^{-1} + 1}{1 - (Z_i)^{-2}} - s_l\right]}$$

And making

$$l := 0 .. n - 1 \qquad BB_{0,l} := \frac{-2\cdot\alpha\cdot\beta}{\beta - s_l}$$

$$AA_l := -1 \qquad BB_{1,l} := \frac{\beta + s_l}{\beta - s_l}$$

$$comp := \prod_{l=0}^{n-1}\frac{1}{\beta - s_l}$$

$$HD1Z_i := \prod_{l=0}^{n-1}\left[\frac{1}{\beta - s_l}\cdot\frac{(Z_i)^2 + AA_l}{(Z_i)^2 + BB_{0,l}\cdot(Z_i) + BB_{1,l}}\right]$$

Then, we have

$$A_{2\cdot l} := \sqrt{-AA_l} \qquad A_{2\cdot l+1} := -\sqrt{-AA_l}$$

IIR Filters Results

Applying the formulas listed above, we are going to show, the coefficients, the expression of H(z), the structure diagram and the algorithm for the simulation for each filter. Finally, we are going to show simultaneously, the magnitude and phase response for both designed filters.

- ***Butterworth Filter***

We will take the n values that belong to:

$$H\ s\ = \frac{1}{\frac{s}{w_0}+1\ \ \frac{s}{w_1}+1\ \ \frac{s}{w_2}+1\ \ ..(\frac{s}{w_{n-1}}+1)}$$

And then, it will be necessary to accommodate the expression like this:

$$H\ s\ = \frac{w_0\ w_1\ w_2\ ...\ w_{n-1}}{s+w_0\ \ s+w_1\ \ s+w_2\ \ ..(s+w_{n-1})}$$

$$H(s) := \frac{1}{\prod_{l=0}^{n-1}(s - s_l)}$$

To find H(z), we will employ

$$s = \beta\ .\frac{z^2 - 2\ \alpha\ z + 1}{z^2 + 1}$$

$$B_{2\cdot l} := \frac{BB_{0,l} + \sqrt{\left(BB_{0,l}\right)^2 - 4\cdot BB_{1,l}}}{2}$$

$$B_{2\cdot l+1} := \frac{BB_{0,l} - \sqrt{\left(BB_{0,l}\right)^2 - 4\cdot BB_{1,l}}}{2}$$

$$l := 0..2\cdot n - 1 \qquad ll := 0..n - 1$$

A =

	0
0	1
1	-1
2	1
3	-1
4	1
5	-1
6	1
7	-1
8	1
9	-1

B =

	0
0	0.2716144233-0.8708805844i
1	-0.2716144233+0.8708805844i
2	0.1547640595-0.7564665537i
3	-0.1547640595+0.7564665537i
4	-0.7138105137i
5	0.7138105137i
6	0.1547640595+0.7564665537i
7	-0.1547640595-0.7564665537i
8	0.2716144233+0.8708805844i
9	-0.2716144233-0.8708805844i

$$HD2Z_i := \left[\frac{\prod_{l=0}^{2\cdot n-1}\left(Z_i + A_l\right)}{\prod_{l=0}^{2\cdot n-1}\left(Z_i + B_l\right)}\right]$$

The next step is to define every coefficient:

$$a_0 := 1 \qquad b_0 := 1$$

$$a_1 := \sum_{r=0}^{2\cdot n-1} A_r \qquad b_1 := \sum_{r=0}^{2\cdot n-1} B_r$$

$$a_2 := \sum_{r=0}^{2\cdot n-2}\sum_{x=r+1}^{2\cdot n-1}\left(A_r\cdot A_x\right)$$

$$b_2 := \sum_{r=0}^{2\cdot n-2}\sum_{x=r+1}^{2\cdot n-1}\left(B_r\cdot B_x\right)$$

$$a_3 := \sum_{l=0}^{2\cdot n-3}\sum_{x=l+1}^{2\cdot n-2}\sum_{m=x+1}^{2\cdot n-1}\left(A_l\cdot A_x\cdot A_m\right)$$

$$b_3 := \sum_{l=0}^{2\cdot n-3}\sum_{x=l+1}^{2\cdot n-2}\sum_{m=x+1}^{2\cdot n-1}\left(B_l\cdot B_x\cdot B_m\right)$$

$$a_4 := \sum_{l=0}^{2\cdot n-4}\sum_{x=l+1}^{2\cdot n-3}\sum_{m=x+1}^{2\cdot n-2}\sum_{y=m+1}^{2\cdot n-1}\left(A_l\cdot A_x\cdot A_m\cdot A_y\right)$$

$$b_4 := \sum_{l=0}^{2\cdot n-4}\sum_{x=l+1}^{2\cdot n-3}\sum_{m=x+1}^{2\cdot n-2}\sum_{y=m+1}^{2\cdot n-1}\left(B_l\cdot B_x\cdot B_m\cdot B_y\right)$$

The coefficients go from a_0 to a_{10} and similarly from b_0 to b_{10}, following the sequence shown above.

Then we have to compensate:

$$comp := \prod_{l=0}^{n-1}\frac{1}{\beta - s_l}$$

$$HD2Z_i := \left[\frac{\prod_{l=0}^{2\cdot n-1}\left(Z_i + A_l\right)}{\prod_{l=0}^{2\cdot n-1}\left(Z_i + B_l\right)}\right]$$

$$l := 0..2\cdot n \qquad a_l := comp\cdot a_l$$

Finally, we obtain the canonic function of the filter, with the corresponding coefficients:

$$HD3Z_i := \frac{\sum_{p=0}^{2\cdot n}\left[a_p\cdot\left(Z_i\right)^{-p}\right]}{1 + \sum_{p=1}^{2\cdot n}\left[b_p\cdot\left(Z_i\right)^{-p}\right]}$$

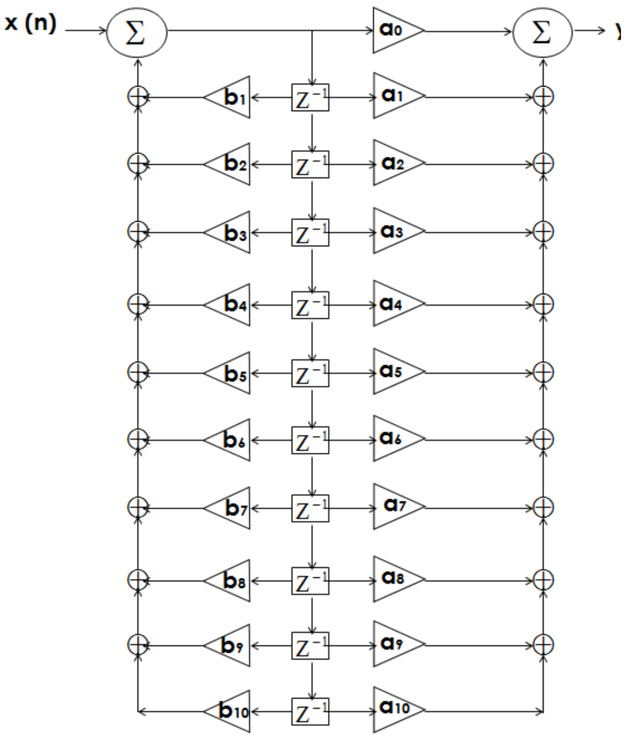
$a_l =$

	0
0	0.0012825811
1	0
2	-0.0064129054
3	0
4	0.0128258108
5	0
6	-0.0128258108
7	0
8	0.0064129054
9	0
10	-0.0012825811

$b_l =$

	0
0	1
1	0
2	2.9754221097
3	-0
4	3.8060181193
5	-0
6	2.5452528683
7	0
8	0.8811300754
9	0
10	0.1254306222

The structure diagram, and the algorithm for the simulation are:



$$y(n) := \sum_{l=0}^{10} (a(n)\cdot x(n-l)) - \sum_{l=1}^{10} (b(n)\cdot y(n-l))$$

- ***Chebyshev I Filter***

We will take the n values that belong to:

$$H\ s\ = \frac{1}{\frac{s}{w_0}+1\ \ \frac{s}{w_1}+1\ \ \frac{s}{w_2}+1\ \ ..(\frac{s}{w_{n-1}}+1)}$$

And then, it will be necessary to accommodate the expression like this:

$$H\ s\ = \frac{w_0\, w_1\, w_2\, ...\, w_{n-1}}{s+w_0\ \ s+w_1\ \ s+w_2\ \ ..(s+w_{n-1})}$$

$$H(s) := \frac{1}{\prod_{l=0}^{n-1} (s - s_l)}$$

To find H(z), we will employ

$$s = \beta\,.\frac{z^2 - 2\,\alpha\,z + 1}{z^2+1}$$

$$\beta := \cot\left[\left(\frac{Fd1 - fd1}{Fs}\right)\cdot\pi\right]$$

$$H(z) := \frac{1}{\prod_{l=0}^{n-1}\left[\left(\beta\cdot\frac{z^{-2} - 2\cdot\alpha\cdot z^{-1} + 1}{1 - z^{-2}}\right) - s_l\right]}$$

Then, we have to define many equations to find the coefficients:

$$i := 0, 1 .. 2000 \qquad \sigma := 0.0000000001$$

$$ESC := 1 \qquad F_i := \frac{i}{2000}\cdot Fs\cdot ESC$$

$$S_i := \sigma + i\cdot 2\cdot\pi\cdot F_i \qquad Z_i := \exp(S_i\cdot T)$$

$$HS_i := \frac{1}{\prod_{l=0}^{n-1} (S_i - s_l)} \qquad HSDB_i := 20\cdot\log\left(\left|\frac{HS_i}{HS_0}\right|\right)$$

$$HSDB(i) := HSDB_i$$

$$HD0Z_i := \frac{1}{\prod_{l=0}^{n-1}\left[\beta\cdot\frac{(Z_i)^{-2} - 2\cdot\alpha\cdot(Z_i)^{-1} + 1}{1 - (Z_i)^{-2}} - s_l\right]}$$

And making

$$l := 0 .. n-1 \qquad BB_{0,l} := \frac{-2\cdot\alpha\cdot\beta}{\beta - s_l}$$

$$AA_l := -1 \qquad BB_{1,l} := \frac{\beta + s_l}{\beta - s_l}$$

$$comp := \prod_{l=0}^{n-1} \frac{1}{\beta - s_l}$$

$$HD1Z_i := \prod_{l=0}^{n-1}\left[\frac{1}{\beta - s_l}\cdot\frac{(Z_i)^2 + AA_l}{(Z_i)^2 + BB_{0,l}\cdot(Z_i) + BB_{1,l}}\right]$$

Then, we have

$$A_{2\cdot l} := \sqrt{-AA_l} \qquad A_{2\cdot l+1} := -\sqrt{-AA_l}$$

$$B_{2\cdot l} := \frac{BB_{0,l} + \sqrt{(BB_{0,l})^2 - 4\cdot BB_{1,l}}}{2}$$

$$B_{2\cdot l+1} := \frac{BB_{0,l} - \sqrt{(BB_{0,l})^2 - 4\cdot BB_{1,l}}}{2}$$

A =

	0
0	1
1	-1
2	1
3	-1
4	1
5	-1
6	1
7	-1
8	1
9	-1

B =

	0
0	0.269974882-0.917485868i
1	-0.269974882+0.917485868i
2	-0.9074270314i
3	0.9074270314i
4	0.269974882+0.917485868i
5	-0.269974882-0.917485868i
6	0.1547640595+0.7564665537i
7	-0.1547640595-0.7564665537i
8	0.2716144233+0.8708805844i
9	-0.2716144233-0.8708805844i

$$HD2Z_i := \left[\frac{\prod_{l=0}^{2\cdot n-1}(Z_i + A_l)}{\prod_{l=0}^{2\cdot n-1}(Z_i + B_l)}\right]$$

The next step is to define every coefficient:

$$a_0 := 1 \qquad b_0 := 1$$

$$a_1 := \sum_{r=0}^{2\cdot n-1} A_r \qquad b_1 := \sum_{r=0}^{2\cdot n-1} B_r$$

$$a_2 := \sum_{r=0}^{2\cdot n-2}\sum_{x=r+1}^{2\cdot n-1}(A_r\cdot A_x)$$

$$b_2 := \sum_{r=0}^{2\cdot n-2}\sum_{x=r+1}^{2\cdot n-1}(B_r\cdot B_x)$$

$$a_3 := \sum_{l=0}^{2\cdot n-3}\sum_{x=l+1}^{2\cdot n-2}\sum_{m=x+1}^{2\cdot n-1}(A_l\cdot A_x\cdot A_m)$$

$$b_3 := \sum_{l=0}^{2\cdot n-3}\sum_{x=l+1}^{2\cdot n-2}\sum_{m=x+1}^{2\cdot n-1}(B_l\cdot B_x\cdot B_m)$$

$$a_4 := \sum_{l=0}^{2\cdot n-4}\sum_{x=l+1}^{2\cdot n-3}\sum_{m=x+1}^{2\cdot n-2}\sum_{y=m+1}^{2\cdot n-1}(A_l\cdot A_x\cdot A_m\cdot A_y)$$

$$b_4 := \sum_{l=0}^{2\cdot n-4}\sum_{x=l+1}^{2\cdot n-3}\sum_{m=x+1}^{2\cdot n-2}\sum_{y=m+1}^{2\cdot n-1}(B_l\cdot B_x\cdot B_m\cdot B_y)$$

The coefficients go from a_0 to a_6 and similarly from b_0 to b_6, following the sequence shown above. Then we have to compensate:

$$comp := \prod_{l=0}^{n-1}\frac{1}{\beta - s_l}$$

$$HD2Z_i := \left[\frac{\prod_{l=0}^{2\cdot n-1}(Z_i + A_l)}{\prod_{l=0}^{2\cdot n-1}(Z_i + B_l)}\right]$$

$$l := 0..2\cdot n \qquad a_l := comp\cdot a_l$$

Finally, we obtain the canonic function of the filter, with the corresponding coefficients:

$$HD3Z_i := \frac{\sum_{p=0}^{2\cdot n}\left[a_p\cdot(Z_i)^{-p}\right]}{1 + \sum_{p=1}^{2\cdot n}\left[b_p\cdot(Z_i)^{-p}\right]}$$

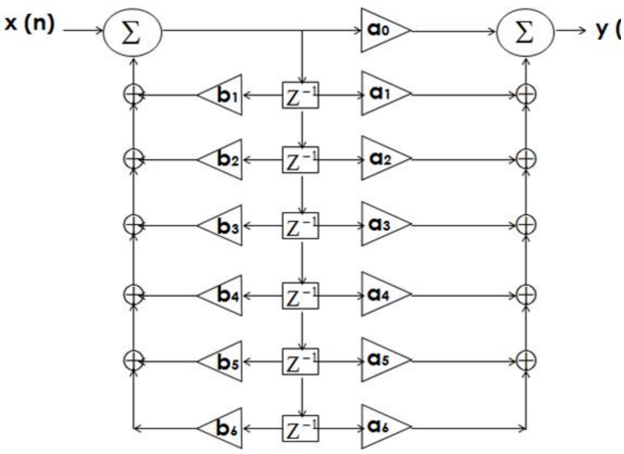
$a_1 =$

	0
0	0.0263829105
1	0
2	-0.0791487316
3	0
4	0.0791487316
5	0
6	-0.0263829105

$b_1 =$

	0
0	1
1	0
2	2.3612115795
3	0
4	2.1028663418
5	0
6	0.6888889412

The structure diagram, and the algorithm for the simulation are:



$$y(n) := \sum_{l=0}^{6} (a(n)\cdot x(n-l)) - \sum_{l=1}^{6} (b(n)\cdot y(n-l))$$

Finally, we will compare the magnitude and phase response of both designed filters, but before that, we will show the correspondent formulas to get these responses for some frequencies.

The frequency response is defined by:

$$HDZDB_i := 20\cdot\log\left(\left|\frac{|HD3Z_i|}{|HD3Z_{500}|}\right|\right)$$

$$\phi HDZ(i) := \arg(HD3Z_i)\cdot\frac{180}{\pi}$$

$$HDZDB(i) := HDZDB_i$$

$$\phi HDZ(i) := \arg(HD3Z_i)\cdot\frac{180}{\pi}$$

$$\phi 1HDZ(i) := \text{if}[\phi HDZ(i) > 0, (\phi HDZ(i) - 360), \phi HDZ($$

Now calculating some attenuation both filters:

- ***Butterworth Filter***

$F_0 = 0$ $\quad HDZDB(0) = -394.3642869112$

$F_{100} = 50000$ $\quad HDZDB(100) = -97.6447922925$

$F_{200} = 100000$ $\quad HDZDB(200) = -62.6962944093$

$F_{300} = 150000$ $\quad HDZDB(300) = -34.9498897368$

$F_{400} = 200000$ $\quad HDZDB(400) = -3.0102999566$

$F_{500} = 250000$ $\quad HDZDB(500) = 0$

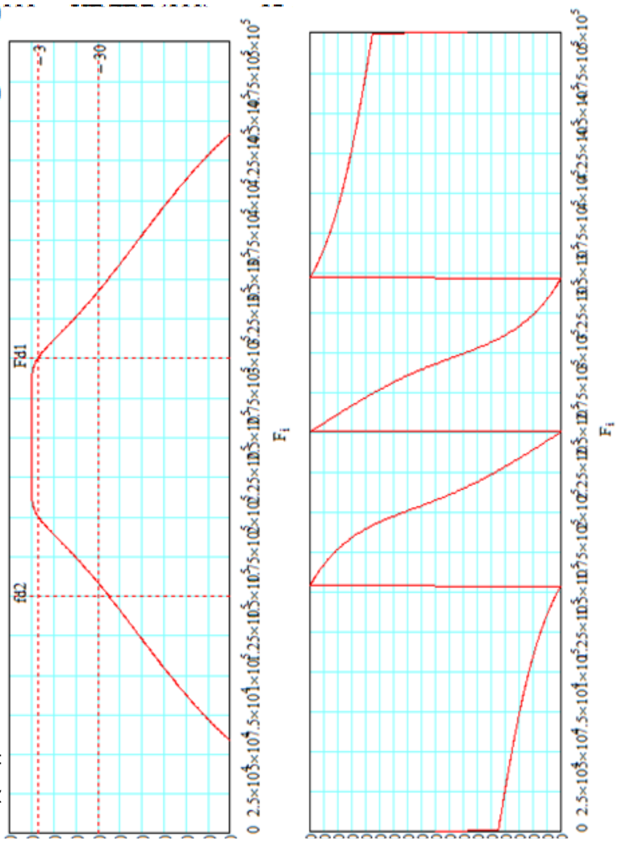
$F_{600} = 300000$ $\quad HDZDB(600) = -3.0102999566$

$F_{700} = 350000$ $\quad HDZDB(700) = -34.9498897368$

$F_{800} = 400000$ $\quad HDZDB(800) = -62.6962944093$

$F_{900} = 450$

$F_{1000} = 50$



F

respo

- ***Chebyshev I Filter***

$F_0 = 0$ HDZDB(0) = −377.0176863198

$F_{100} = 50000$ HDZDB(100) = −70.5551632065

$F_{200} = 100000$ HDZDB(200) = −49.2881852137

$F_{300} = 150000$ HDZDB(300) = −31.6016829296

$F_{400} = 200000$ HDZDB(400) = −3.0102999566

$F_{500} = 250000$ HDZDB(500) = 0

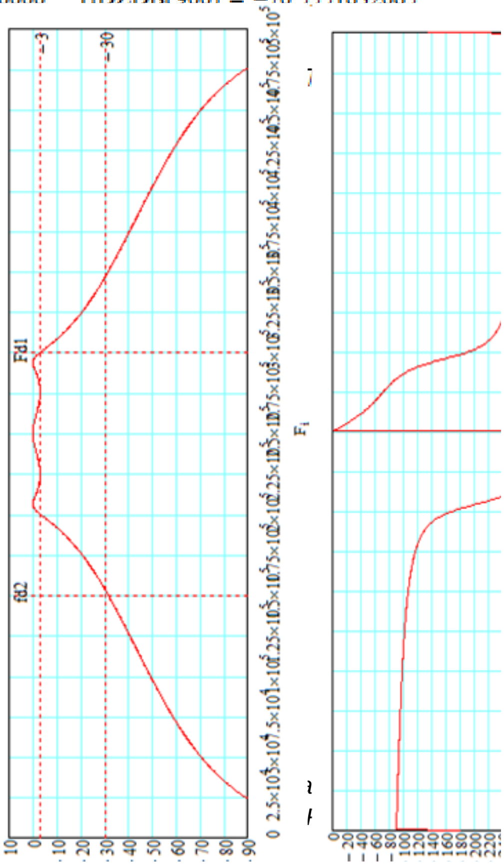
$F_{600} = 300000$ HDZDB(600) = −3.0102999566

$F_{700} = 350000$ HDZDB(700) = −31.6016829296

$F_{800} = 400000$ HDZDB(800) = −49.2881852137

$F_{900} = 450000$ HDZDB(900) = −70.5551632065

$F_{1000} = 5$

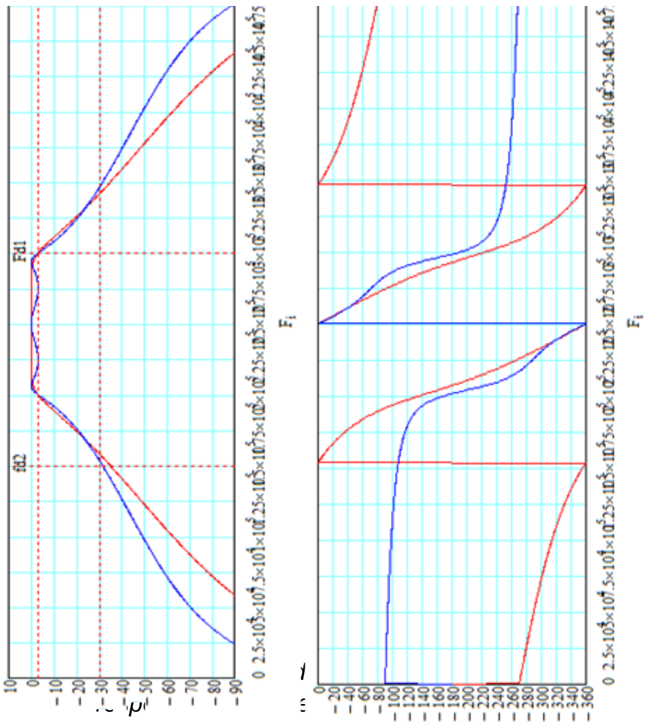


resp

From Figure 2 and Figure 3, we can see that Butterworth and Chebyshev I filter follow the characteristics specified for the design; we can see the cut frequencies and the stop frequencies required.

Finally, we will compare the two filter responses, in magnitude and phase:

- The red graphic shows the Butterworth filter response.
- The blue graphic shows the Chebyshev I filter response.



5 CONCLUSIONS

FIR Filters Results

The response of all the filters, in terms of phase, is lineal and resembles each other. So, since these filters are FIR filters, they don't have phase distortion, that it's very important. Talking about the amplitude, the best filter of this four, is the Kaiser filter with β = 4, because this filter attenuate best the frequencies that are not in the band pass, also this filter is the least distorts the amplitude, so these two reasons make this filter the best in comparison with the other three.

IIR Filters Results

The responses of both filters, in terms of phase, are a little different, because the Chebyshev I filter, have phase distortion, and the Butterworth phase response have some linear parts. So, since these filters are IIR filters, they have phase distortion, that it's a disadvantage. Talking about the amplitude, the best filter of this two is the Chebyshev I filter, with this filter results in a fall of frequency response more pronounced at low frequencies and it allow a ripple in the pass band as shown in the magnitude response above.

In conclusion comparing the two types of filters, after seeing the magnitude and phase responses, we see that IIR filters are characterized by greater attenuation to frequencies that are not in the pass band, besides the most important feature of these filters is that the phase response is linear. One advantage of IIR filters is that we will need smaller number of coefficients for the specified design features, but today that's no problem because the microprocessor can use a large amount of coefficients. Then the best option is to use FIR filters.

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